Observation of a Heteroclinic Tangency Crisis in an NMR-laser System

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Symbolic-dynamical encoding of chaotic dynamics yields a precise quantitative characterization of the temporal scaling behavior at crisis points. A complete analysis in the heteroclinic tangency case is presented for an NMR-laser.

Key words: Laser chaos, Crisis-induced intermittency, Symbolic dynamics.

Strange attractors may undergo sudden structural changes upon variation of an external parameter p. These transitions, called crises in [1], are determined by the collision between the unstable manifold of some periodic orbit A and the stable manifold either of the same orbit A (homoclinic crisis) or of another orbit B, with the same period as A (heteroclinic crisis). In both cases the strange attractor, which coincides with (the closure of) the unstable manifold of A, widens over a region of phase space which was "empty" for p smaller than the crisis value p_c. Just above crisis, the dynamics consists of intermittent "bursting" between the two subsets of phase space. This behavior is characterized by the average time $\tau(p)$ elapsing between two consecutive bursts. In [1] it has been conjectured that $\tau(p) \sim |p-p_c|^{-\gamma}$ for $p \approx p_c$, where the critical exponent γ has been predicted to lie in the interval $1/2 \leqq \gamma \leqq 3/2$ (corresponding to the strongly dissipative and conservative limits, respectively).

However, a precise definition of the two domains in phase space ("inside" and "outside" or "core" and "transient") is not easy to achieve when the equations of motion (and, therefore, the invariant manifolds) are not known. This is particularly evident with experimental data, so far analyzed just by taking an arbitrary partition in the embedding space. We resolve this ambiguity by constructing an approximation to a generating partition Δ and by identifying the "outer" region with the element of the refinement of Δ with the same label as the "new" symbolic sequence appearing in the dynamics. In this way, we have a clear cut definition of the times spent in the two regions. We apply this technique to experimental data produced by a parametrically modulated NMR-laser. The critical exponent γ has been estimated both for experimental and model data in correspondence of the heteroclinic crisis of a period-3 orbit.

The NMR-laser [2] activity is provided by nuclear ^{27}Al spins in a ruby crystal placed in a static magnetic field at liquid-helium temperature. The spin population inversion is obtained by means of dynamical nuclear polarization and the laser action by enclosing the active medium in a cavity (here, with modulated quality factor $Q(t) = Q_0(1 + p\cos\omega t)$). The laser output is a voltage, proportional to the transverse nuclear magnetization. An adequate description of the laser dynamics can be achieved with a generalization of the Maxwell-Bloch phenomenological model [2, 3].

Since the dynamics is non-autonomous, Poincaré sections can be readily obtained for both experimental and numerical data by sampling the trajectories synchronously with the forcing term and by embedding them in a 3- to 16-dimensional space (in fact, the dimension of the attractor is about 2.5). A (binary) generating partition has been approximated by attributing a different symbolic label to each point belonging to an unstable periodic orbit [3]. The only sequence which is forbidden below crisis ($S = 00$) becomes allowed when the new period-3 orbit B (labelled 001) starts belonging to the closure of the broadened attractor (the other period-3 orbit having label 011), for $p \geqq p_c = 0.01802$. Therefore, we consider a point to

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lie in the "outer" region if it produces the sequence 00: the re-immission into the "inner" part is detected when either a 1 or a 01 follows 001. Accordingly, we have estimated the characteristic times $\tau(p)$ for 14 different values of p and determined the critical exponent γ from the corresponding log-log plots. We obtained $\gamma_e = 1.02 \pm 0.05$ and $\gamma_m = 1.10 \pm 0.05$ for experiment and model, respectively.

The values of γ lie in the predicted interval and are consistent with the value 2.5 for the dimension, since the latter is well approximated by $2 + \lambda_1/|\lambda_2|$ and, in the heteroclinic case, $\gamma = 1/2 + \lambda_1/|\lambda_2|$, where λ_i is the i-th Lyapunov exponent of orbit A [1].

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